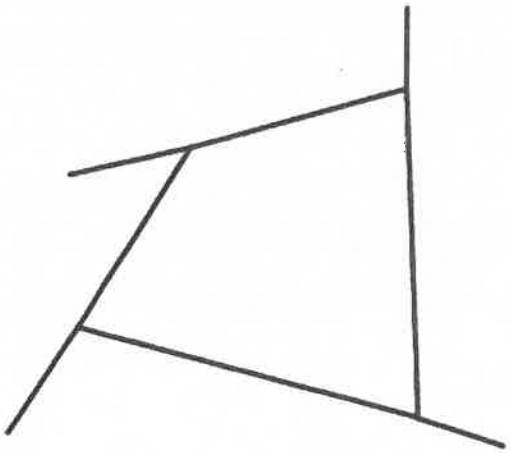
Completed

Unit 5: Polygons

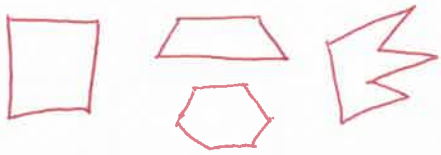
		Initial Score	*Updated Score*
1	I can find the interior and exterior angles of a polygon.		
2	I can identify and use the properties of a parallelogram.		
3	I can identify and use the properties of all quadrilaterals.		



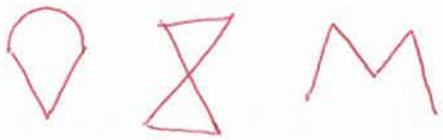
(1) **Vocabulary:**

Polygon:
A closed plane figure formed by segments. Each side intersects exactly 2 other sides. No two sides are collinear

Examples of Polygons:



Examples of Non-polygons:



Number of sides	Name	Picture
3	Triangle	
4	Quadrilateral	
5	Pentagon	
6	Hexagon	
7	Heptagon	
8	Octogon	
9	Nonagon	
10	Decagon	

Sum of Interior Angles

The sum of the measures of the interior angles of a convex polygon is

$$S_{int} = (n-2)\cdot 180$$

n = # of sides / angles

Ex) Find the sum of the interior angles of a pentagon.

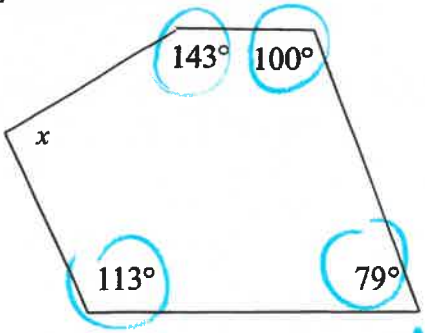
↳ 5 sides

$S = (5-2)\cdot 180$

$S = 3 \cdot 180$

$S = 540$

Ex) Find the value of x:



$540 - 143 - 100 - 79 - 113$

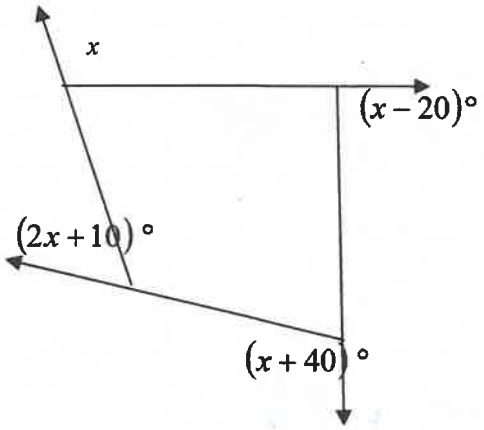
$105°$

Sum of Exterior Angles

The sum of the measures of the exterior angles of any convex polygon is equal to

$$S_{ext} = 360$$

Ex) Find the value of x:



$x + (x-20) + (x+40) + (2x+10) = 360$

$5x + 30 = 360$

$5x = 330$

$x = 65$

Each Interior Angle

For a regular polygon, each interior angle is

$$\frac{(n-2)\cdot 180}{n}$$

(n = # sides)

Ex) Find the measure of each interior angle in a regular 20-gon.

↳ $n = 20$

$$\frac{(20-2)\cdot 180}{20}$$

$$\frac{(18)\cdot 180}{20}$$

$$\frac{3240}{20} = 162^\circ$$

Each Exterior Angle

For a regular polygon, each exterior angle is

$$\frac{360}{n}$$

Ex) Find the measure of each exterior angle in a regular nonagon.

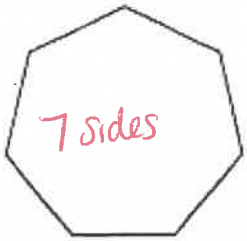
↳ 9 sides

$n = 9$

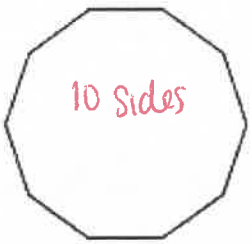
$$\frac{360}{9} = 40^\circ$$

Polygon Angle Practice:

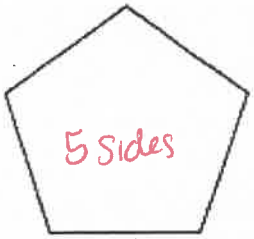
Find the interior angle sum for each polygon. Round your answer to the nearest tenth if necessary.

1) 

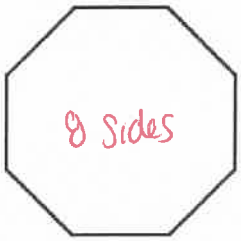
$(7-2)180 = 900°$

2) 

$(10-2)180 = 1440°$

3) 

$(5-2)180 = 540°$

4) 

$(8-2)180 = 1080°$

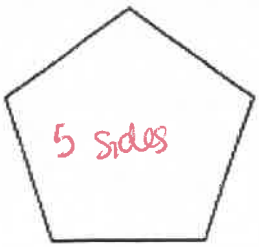
5) regular quadrilateral $360°$

6) regular 18-gon $2880°$

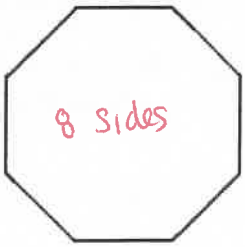
7) regular dodecagon $1800°$

8) regular 15-gon $2340°$

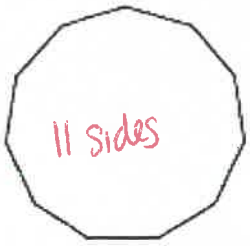
Find the measure of one interior angle in each polygon. Round your answer to the nearest tenth if necessary.

1) 

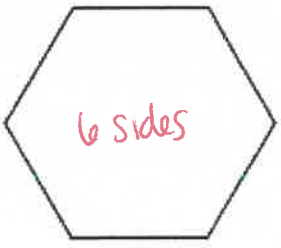
$\frac{(5-2)180}{5} = 108°$

2) 

$\frac{(8-2)180}{8} = 135°$

3) 

$\frac{(11-2)180}{11} = 147.3°$

4) 

$\frac{6-2(180)}{6} = 120°$

5) regular 24-gon $165°$

6) regular quadrilateral $90°$

7) regular 23-gon $164.3°$

8) regular 16-gon $157.5°$

(5)

Find the measure of one exterior angle in each polygon. Round your answer to the nearest tenth if necessary.

1) 10 sides

$\frac{360}{10} = 36°$

2) 7 sides

$\frac{360}{7} = 51.4°$

3) 8 sides

$\frac{360}{8} = 45°$

4) 5 sides

$\frac{360}{5} = 72°$

5) egular 13-gon $27.7°$

6) regular 16-gon $22.5°$

7) egular 20-gon $18°$

8) regular 23-gon $15.7°$

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1) Name the polygon based on the number of sides

hexagon

2) Name the polygon using the angles in two different ways

polygon ABCDEF / polygon FEDCBA

3) Name a pair of consecutive angles

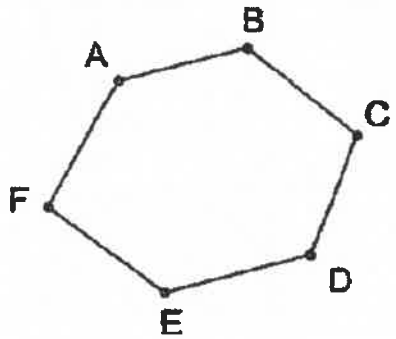
$\angle E$ & $\angle D$

4) Name a pair of adjacent sides

$\overline{AB}$ & $\overline{BC}$

5) Name a pair of opposite vertices

B & E



Parallelogram:

Properties of parallelograms:

1.) Opposite sides parallel

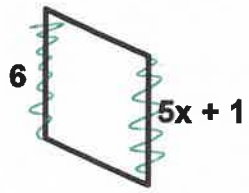
2.) Opposite sides equal

3.) Opposite angles equal

4.) Consecutive angles Supplementary (180°)

5.) Diagonals bisect each other

Examples: Solve for the missing variable.

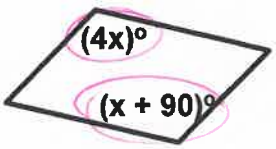


$5x+1=6$

$5x=5$

$x=1$

Opposite sides equal

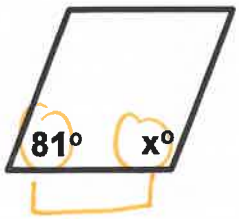


$4x = x+90$

$3x = 90$

$x=30$

opposite angles equal

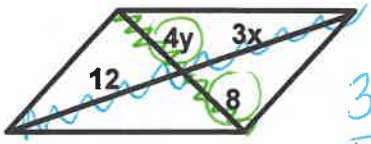


180

$x+81=180$

$x=99$

Consecutive angles Supp



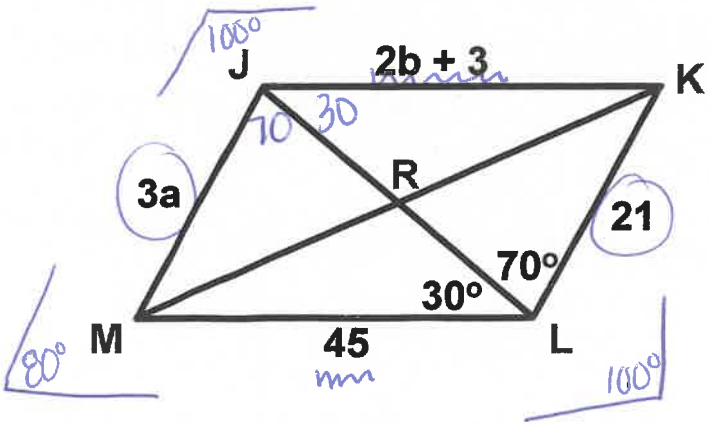
$3x=12$

$x=4$

$4y=8$

$y=2$

Use parallelogram JLKM to find each measure or value.



$3a=21$

$a=7$

$2b+3=45$

$2b=42$

$b=21$

a = 7

b = 21

m∠ MJK = 100°

m∠ JML = 80°

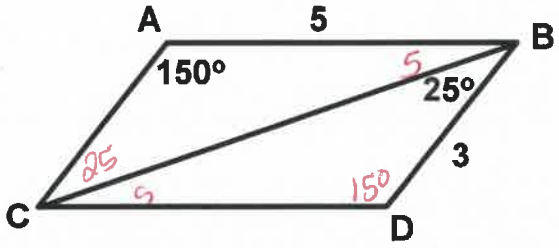
m∠ KJL = 30°

(7)

Parallelogram Practice:

Find each indicated measure.

1. ABCD is a parallelogram.

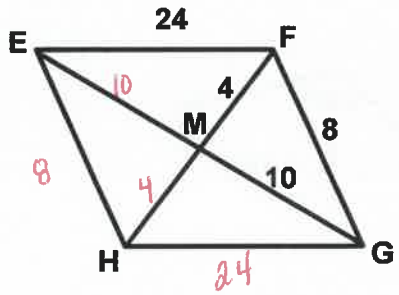


AC = 3 CD = 5

m∠D = 150° m∠ABC = 5°

m∠ACB = 25° m∠BCD = 5°

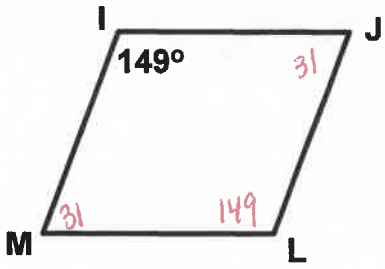
2. EFGH is a parallelogram.



EH = 8 HG = 24

MH = 4 EM = 10

3. IJLM is a parallelogram.

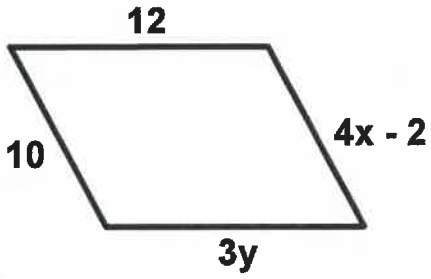


m∠J = 31°

m∠L = 149°

m∠M = 31°

4. Find x and y.



$4x-2=10$

$4x=12$

$x=3$

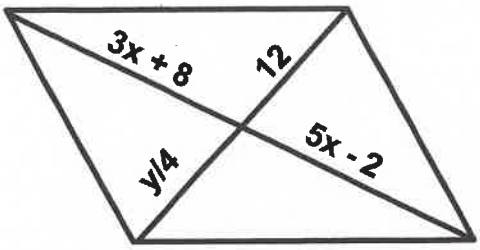
$3y=12$

$y=4$

x = 3

y = 4

5. Find x and y.



$3x+8 = 5x-2$

$10 = 2x$

$x = 5$

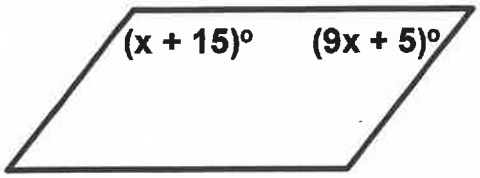
$(4)\frac{y}{4} = 12\ (4)$

$y = 48$

x = 5

y = 48

6. Find x.



$x+15+9x+5 = 180$

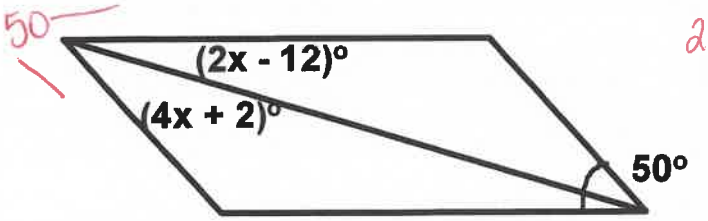
$10x+20 = 180$

$10x = 160$

$x = 16$

x = 16

7. Find x.



$2x-12+4x+2 = 50$

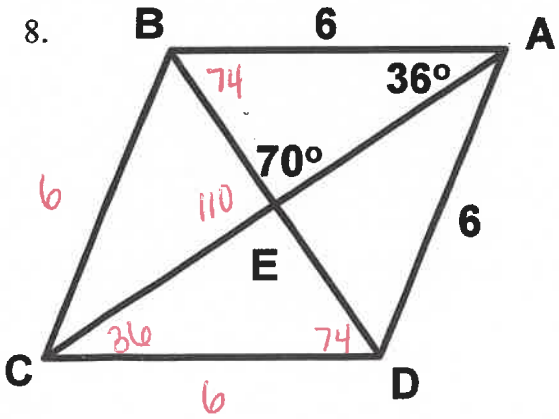
$6x-10 = 50$

$6x = 60$

$x = 10$

x = 10

8.



BC = 6

CD = 6

m ∠ CED = 70°

m ∠ ABE = 74°

m ∠ BEC = 110°

m ∠ ACD = 36°

m ∠EDC = 74°

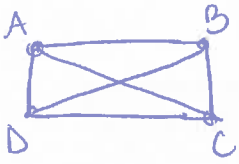
(9) **Rectangle:**

Properties of Rectangles:

- All the properties of a parallelogram (there are 5)

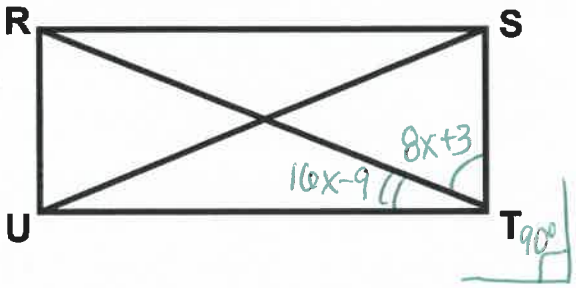
  PLUS

1) 4 right angles 

2) Diagonals are equal 

AC = BD

Example 1: Find the value of x if m∠ STR = 8x + 3 and m∠UTR = 16x – 9



$8x+3+16x-9=90$

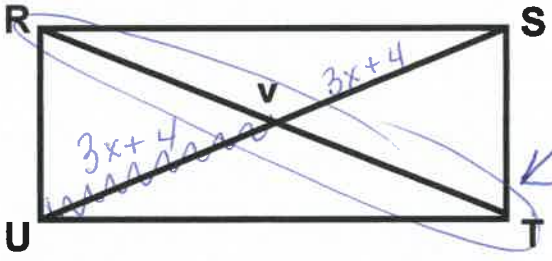
$24x-6=90$

$24x=96$

$x=4$

Corner angle = 90°

Example 2: Find the value of x if UV = 3x + 4 and RT = 7x – 2.



$(3x+4)+(3x+4)=7x-2$

$6x+8=7x-2$

$10=x$

Diagonals are equal

Rectangle Practice:

10

Use rectangles ABCD and the given information to solve each problem.

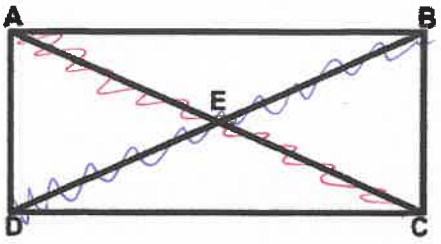
1) If $AC = 4x - 60$ and $BD = 30 - x$, find BD.

$4x - 60 = 30 - x$

$5x = 90$

$x = 18$

$BD = 12$



2) If $AC = 4X - 60$ and $AE = x + 5$, find EC.

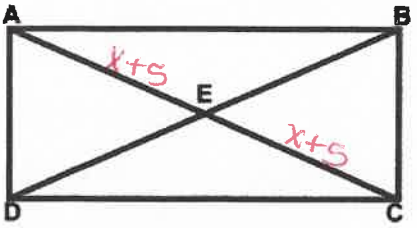
$4x - 60 = x + 5 + x + 5$

$4x - 60 = 2x + 10$

$2x = 70$

$x = 35$

$EC = 40$



3) If $m\angle BAC = 4x + 5$ and $m\angle CAD = 5x - 14$, find $m\angle CAD$.

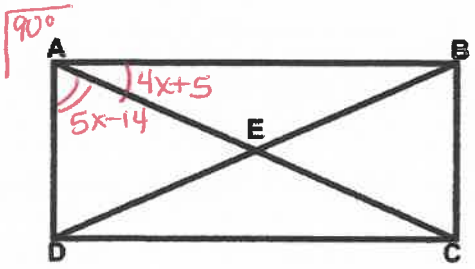
$4x + 5 + 5x - 14 = 90$

$9x - 9 = 90$

$9x = 99$

$x = 11$

$m\angle CAD = 41°$



4) If $AE = 2x + 3$ and $BE = 12 - x$, find BD.

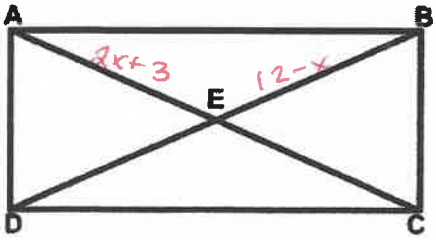
$2x + 3 = 12 - x$

$3x = 9$

$x = 3$

$BD = 9 + 9$

$BD = 18$



5) $m\angle BAC = 3x + 5$ and $m\angle ACD = 40 - 2x$. Find $m\angle AED$.

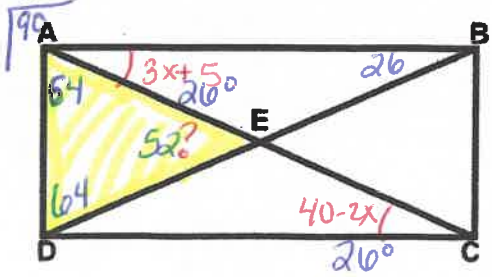
$3x + 5 = 40 - 2x$

$5x = 35$

$x = 7$

$180 - 64 - 64 = 52$

$m\angle AED \rightarrow 52$

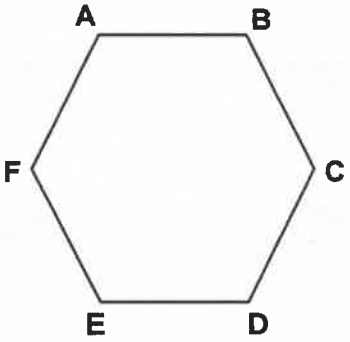


STOP

(11)

Review Day:

Use the polygon below to answer each question.

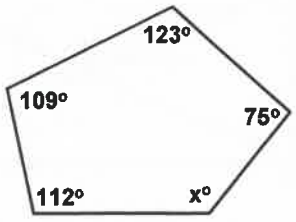


1. Name the polygon hexagon
2. Name an angle consecutive to ∠D: ∠E or ∠C
3. Name a side adjacent to $\overline{AB}$: $\overline{AF}$ or $\overline{BC}$
4. Name a vertex opposite of ∠C: ∠F

5. Given each regular polygon, find each measure.

Polygon	*# of sides*	*Sum of the interior angles*	*One interior angle*	*Sum of the exterior angles*	*One exterior angle*
Quadrilateral	4	360	90	360	90
Octagon	8	1,080	135	360	45
11-gon	11	1620	147.3	360	32.7

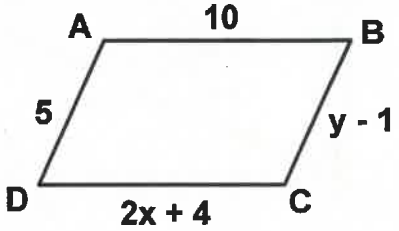
6. Solve for x.



(n-2)180
(5-2) 180
(3) 180
540

x = 121

7. Given parallelogram ABCD, solve for x and y.



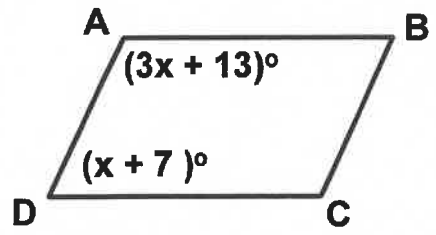
2x+4=10
2x=6
x=3

y-1=5
y=6

x = 3

y = 6

8. Given parallelogram ABCD, find each value.



$3x+13+x+7=180$

$4x+20=180$

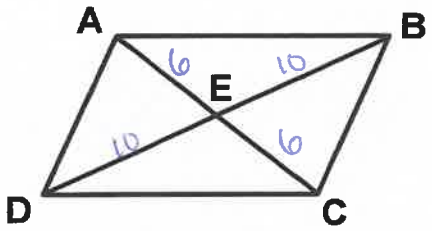
$4x=160$

$x=40$

x = 40 $m\angle A$ = 133°

$m\angle B$ = 47° $m\angle C$ = 133°

9. Given: AE = x + 3, EC = 7x – 15, and DB = 20 in parallelogram ABCD.



$x+3=7x-15$

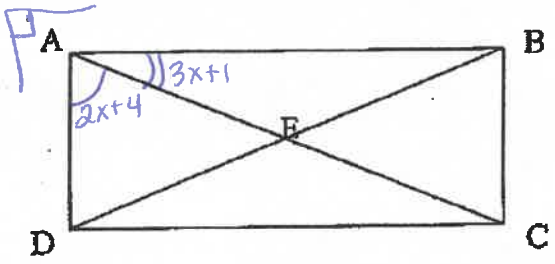
$18=6x$

$3=x$

x = 3 AE = 6

AC = 12 BE = 10

10. If $m\angle DAC = 2x+4$ and $m\angle BAC = 3x+1$, find x.



$2x+4+3x+1=90$

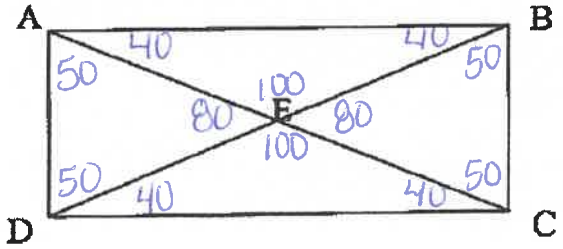
$5x+5=90$

$5x=85$

$x=17$

x = 17

11. If $m\angle EAD = 50°$, find the measure of all the angles in the rectangle.



(13)

Rhombus:

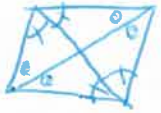
Properties of Rhombi:

- All the properties of a parallelogram (there are 5)

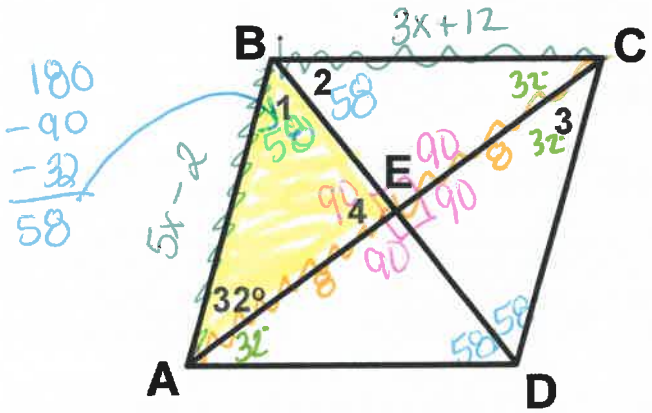
PLUS

1) All sides equal 

2) Diagonals are perpendicular (90°) 

3) Diagonals bisect each corner angle 

Example 1) Find the measure of each numbered angle.



m∠1 = 58° (180° in triangle)

m∠2 = 58° (angle bisected)

m∠3 = 32° (alt. int. angles)

m∠4 = 90° (diag perpendecular)

Example 2) If AE = 8, find AC.

diagonals bisected 8+8 = 16

Example 3) If BC = 3x + 12 and AB = 5x - 2, find x.

all sides equal $3x+12 = 5x-2$

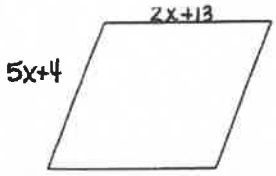
$14 = 2x$

$x = 7$

Rhombus Practice:

Using the properties of rhombi, write and solve an algebraic equation for each picture.

1.



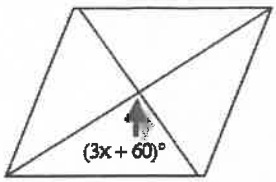
$5x+4 = 2x+13$

$3x = 9$

$x = 3$

$x =$ 3

2.



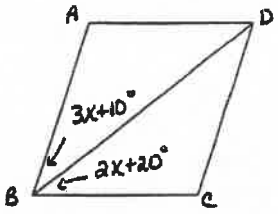
$3x + 60 = 90$

$3x = 30$

$x = 10$

$x =$ 10

3.



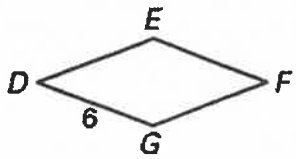
$3x+10 = 2x+20$

$x = 10$

$x =$ 10 $m\angle ABD =$ 40 $m\angle ABC =$ 80

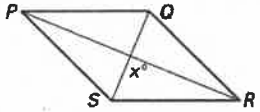
Use each RHOMBUS to find the specified lengths and measures.

4.



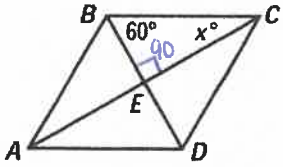
DE = 6 EF = 6 GF = 6

5.



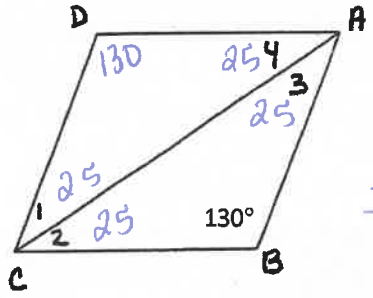
$x =$ 90°

6.



$x =$ 30

7.

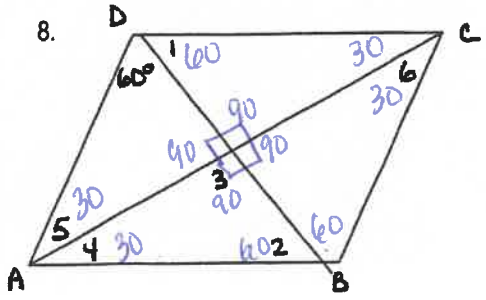


$180 - 130 = 50 \div 2 = 25$

$m\angle D =$ 130 $m\angle DCB =$ 50° $m\angle 1 =$ 25°

$m\angle 2 =$ 25° $m\angle 3 =$ 25° $m\angle 4 =$ 25°

8.



$m\angle 1 =$ 60° $m\angle 2 =$ 60° $m\angle 3 =$ 90

$m\angle 4 =$ 30° $m\angle 5 =$ 30° $m\angle 6 =$ 30°

$m\angle ADC =$ 120° $m\angle DAB =$ 60°

STOP

(15) **Square:**

Properties of Squares:

Parallelogram
1.) Opposite sides parallel
2.) Opposite sides equal
3.) Opposite angles equal
4.) Consecutive angles Supplementary (180°)
5.) Diagonals bisect each other

Rectangle
6.) All right angles
7.) Diagonals equal

Rhombus
8.) All sides equal
9.) Diagonals perpendicular (90°)
10.) Diagonals bisect corner angles

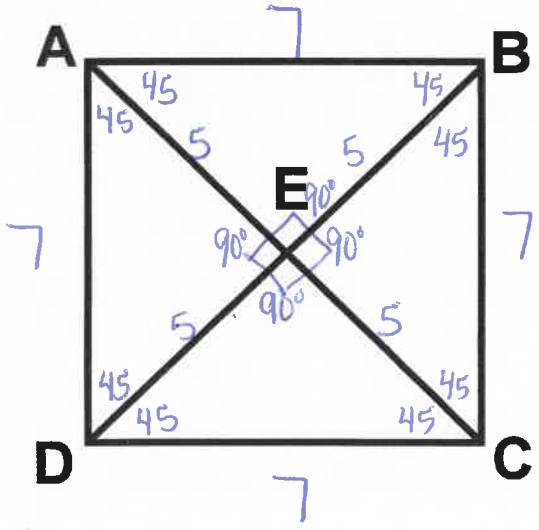
Example 1: If AB = 7, find the length of each side

Each side = 7

Example 2: If AE = 5, find the length of each diagonal.

Diagonal = 5 + 5 = 10

Example 3: Fill in every angle measurement



Square Practice:

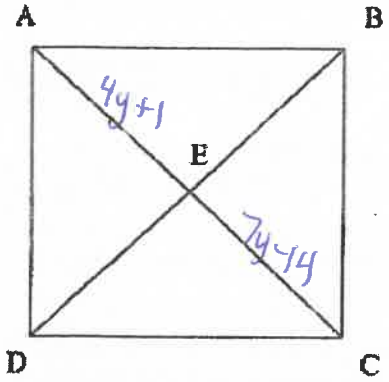
Use square ABCD to find each indicated measure.

1. If $AE = 4y+1$ and $EC = 7y-14$, find y.

$4y+1 = 7y-14$

$15 = 3y$

$y = 5$



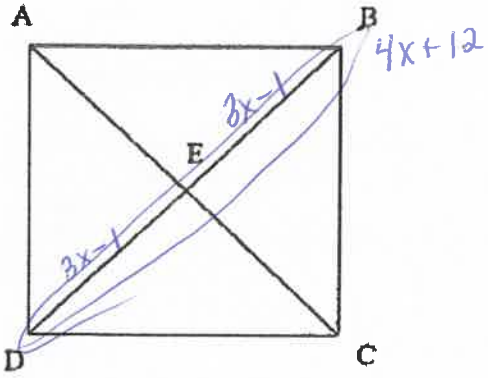
2. If $ED = 3x-1$ and $DB = 4x+12$, find x.

$3x-1+3x-1 = 4x+12$

$6x-2 = 4x+12$

$2x = 14$

$x = 7$

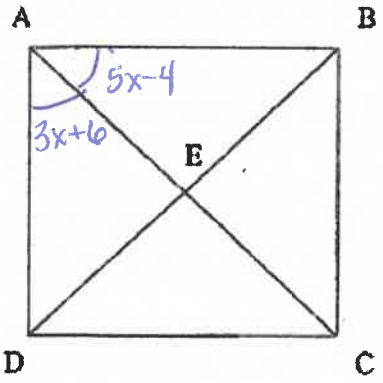


3. If $m\angle DAC = 3x+6$ and $m\angle BAC = 5x-4$, find x.

$3x+6 = 5x-4$

$10 = 2x$

$5 = x$

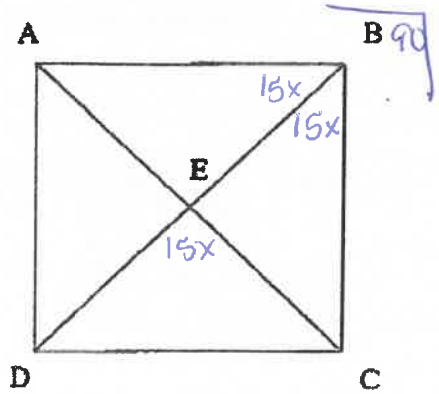


4. If $m\angle EBC = 15x$, find x.

$15x + 15x = 90$

$30x = 90$

$x = 3$



5. If $m\angle DEC = 15x$, find x.

$15x = 90$

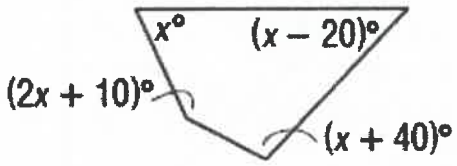
$x = 6$

(17)

Review:

1. Set up an equation to help you solve for x.

x = 65



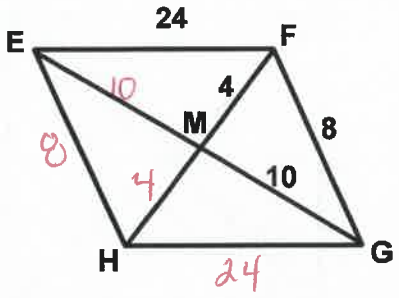
$x + x-20 + x+40 + 2x+10 = 360$

$5x + 30 = 360$

$5x = 330$

$x = 65$

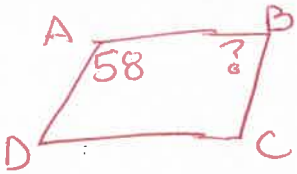
2. EFGH is a parallelogram.



EH = 8 HG = 24

MH = 4 EM = 10

3. In parallelogram *ABCD*, $m\angle A = 58$. Find $m\angle B$. (hint: draw a picture

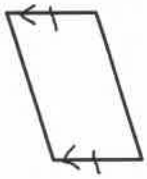


$58 + x = 180$

$x = 122$

4. Determine if each quadrilateral is a parallelogram. If yes, state why.

a.



NO

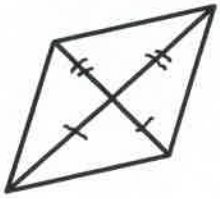
b.

yes - opposite angles equal

c.

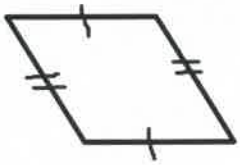
yes - diagonals bisected

d.



NO

e.

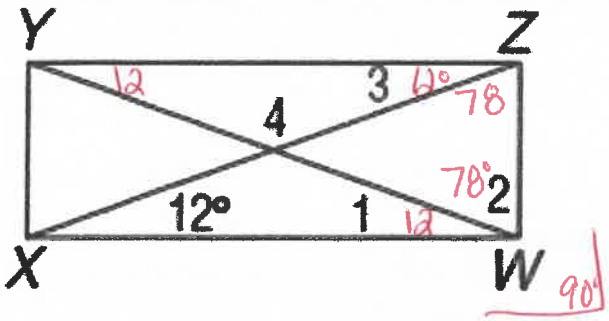


yes - opposite sides equal

f.

NO

5. Given ABCD is a rectangle, solve for every angle measure.



180
− 12
− 12
156

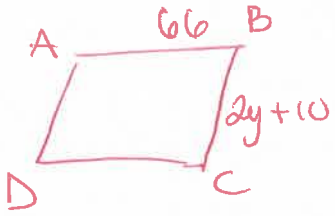
$m\angle 1 =$ 12°

$m\angle 2 =$ 78°

$m\angle 3 =$ 12°

$m\angle 4 =$ 156°

6. In rhombus ABCD, AB= 66 and BC = 2y + 10, find y.

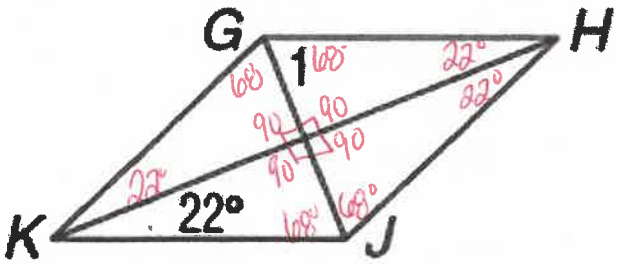


$2y + 10 = 66$

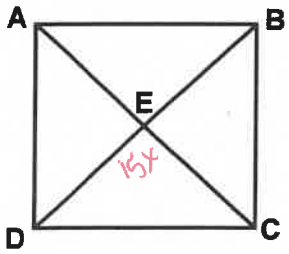
$2y = 56$

$y = 28$

7. Fill in each angle measure for the rhombus below.



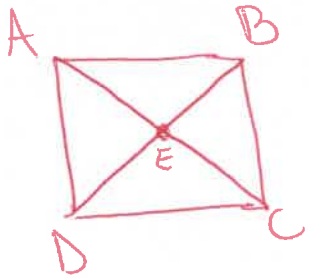
8. Find the value of x if $m\angle$ DEC = 15x.



$15x = 90$

$x = 6$

9. The diagonals of square *ABCD* intersect at *E*. If *AE*= 2*x* + 6 and *BD* = 6*x* - 10, find *AC*.



$2x + 6 + 2x + 6 = 6x - 10$

$4x + 12 = 6x - 10$

$22 = 2x$

$11 = x$

$AC = 56$

(19)

Kite:

Kites are NOT members of the parallelogram family. Therefore, they <u>DO NOT</u> possess the characteristics of a parallelogram.

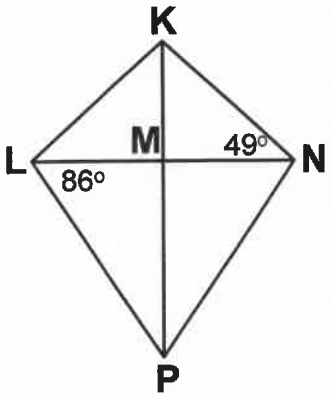
Properties of Kites:

1. No parallel sides
2. Two pairs of consecutive sides congruent
3. One pair of opposite angles congruent
4. The long diagonal bisect the short diagonal
5. The long diagonal bisects one pair of opposite angles
6. The diagonals are perpendicular

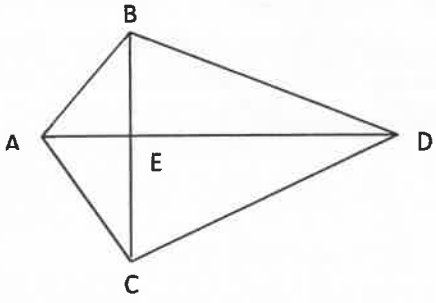
Hint: _____

Examples

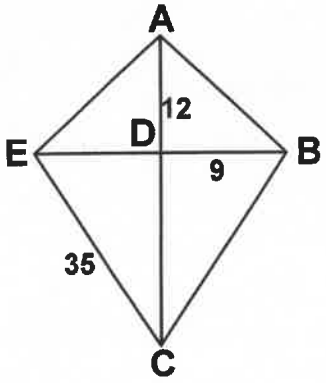
1. Fill in all angles



2. If $m\angle CED = 4x - 10$, find x.



3. Fill in all segment lengths



4. Find the value of and y