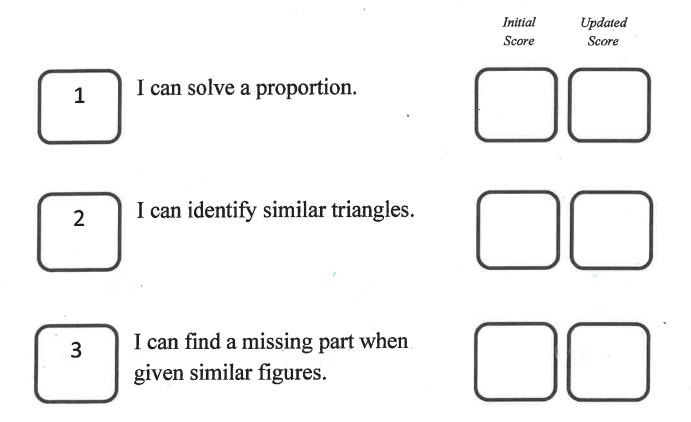
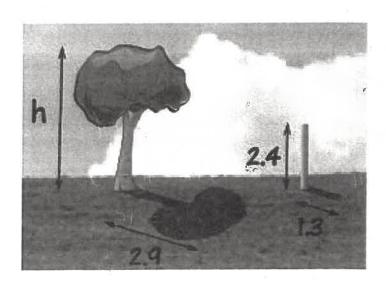
Unit 6: Similarity





• Ratios & Proportions:

Ratio- A comparison of two quantities ex) 3 to 4, 3:4, 3/4

Write a ratio for the following scenarios:

Ex) An archer hits the scoring target 44 times out of 60 attempts.

Ex) Out of 1,846 students in Batavia High School, 47 play baseball.

1846

Proportion - When two ratios (fractions) are equal

** Cross products are equal

6=6

Solve each proportion:

a)
$$\frac{5}{8} = \frac{x}{12}$$

b)
$$\frac{x}{1.12} = \frac{1}{5}$$

c)
$$\frac{6x}{27} = \frac{4}{3}$$

$$5x = 1.12$$

$$18x = 108$$

d)
$$\frac{x+2}{3} = \frac{8}{9}$$

e)
$$\frac{3x-5}{4} = \frac{-5}{7}$$

f)
$$\frac{x-2}{4} = \frac{x+4}{2}$$

$$7(3x-5) = 4(-5)$$

$$2(x-2) = 4(x+4)$$

$$9x + 18 = 24$$

$$21x - 35 = -20$$

$$2x = -20$$

$$X = 0.6$$

$$x = .714$$

$$X = -10$$

Application

- A. Set up a proportion to describe the scenario.
- B. Solve your proportion.
- C. Answer the question including units
- 1. Your new car can drive 495 miles on 16.5 gallons of gas. How far will you make it on 11 gallons of gas?

$$16.5x = 5445$$

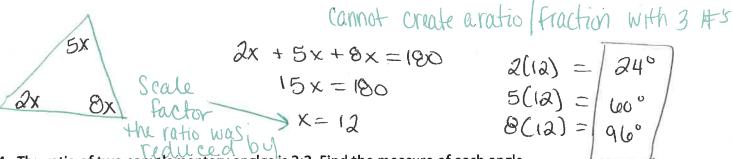
 $x = 330$
 330 miles

2. Suppose the Chicago Cubs won 20 of their first 55 games. If they continue to win at the same rate, how many games will they have won by the end of the season if they play a total of 165 games?

$$55 \times = 3300$$

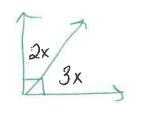
 $\times = 60$
 $\times = 60$

3. In a triangle, the ratio of the measure of the three angles is 2:5:8. Find the measure of each angle.



4. The ratio of two complementary angles is 2:3. Find the measure of each angle.

add to 90



$$2x + 3x = 90$$

 $5x = 90$
 $\Rightarrow x = 18$

$$(2)(10) = 36^{\circ}$$

$$(3)(10) = 54^{\circ}$$

Ratio & Proportion Practice:

Solve for x.

1.
$$\frac{5x}{4} = \frac{35}{8}$$

2.
$$\frac{7}{2} = \frac{x+1}{3}$$

3.
$$\frac{x}{3} = \frac{x+9}{12}$$

$$(x=3.5)$$

$$2x+2=21$$

$$2x=19$$

$$\chi = 9.5$$

$$3x+27=12x$$
 $27=9x$

4. The scale on a map is 2 inches to 35 miles. If the distance between two cities is 152 miles, how long is the segment between the two cities on the map?

$$\frac{2 \text{ in}}{35 \text{ mi}} = \frac{x \text{ in}}{152 \text{ mi}}$$

$$35x = 304$$

 $x = 8.7$



5. The ratio of male students to female students in the drama club at Campbell High School is 3:4. If the number of male students in the club is 18, what is the number of female students?

$$3x = 72$$
$$x = 24$$



6. If GJKL ~ MOPR, find MR, MO, OP, $m \angle$ O, and $m \angle$ G.

$$\frac{9}{0} = \frac{8}{x}$$

$$\frac{9}{0} = \frac{6}{y}$$
 $\frac{9}{0} = \frac{12}{Z}$

$$\frac{9}{6} = \frac{12}{Z}$$

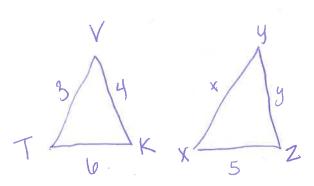
$$MR = 5.\overline{3} \qquad MO = 4$$

$$OP = 8 \qquad m \angle P = 47^{\circ}$$

$$m \angle R = 133^{\circ}$$



7. Given \triangle TVK \sim \triangle XYZ, TV = 3, VK = 4, TK = 6, and XZ = 5. Find XY and ZY.



$$\frac{6}{5} = \frac{3}{X}$$
 $6x = 15$

$$XY = 2.5$$

$$ZY = 3.5$$

$$\frac{6}{5} = \frac{4}{y}$$
 $6y = 20$ $y = 3.3$

Find the measures of the sides or angles of each triangle.

8. The ratio of the measures of the sides is 3:5:7, and its perimeter is 450 centimeters.

$$3x + 5x + 7x = 450$$

 $15x = 450$
 $x = 30$

$$3(30) = 90 \text{ cm}$$

$$5(30) = 150 \text{ cm}$$

$$7(30) = 210 \text{ cm}$$

9. The ratio of the measures of the sides is 5:6:9, and its perimeter is 220 meters.

$$5x + 6x + 9x = 220$$

$$20x = 220$$

$$x = 11$$

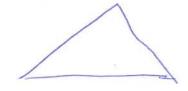
10. The ratio of the measures of the angles is 4:5:6 in a triangle.

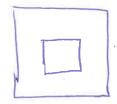
$$4x + 5x + 6x = 180$$
 $15x = 180$
 $x = 12$

6 Similar Figures:

Brainstorm - create two similar figures:







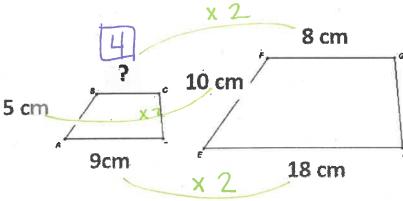
Similar Figures: We the Same Shape, but Change

Sire

- · Angles are equal
- · Sides are proportionate

The length changes by the same scale on all sides. This is called a

scale factor.



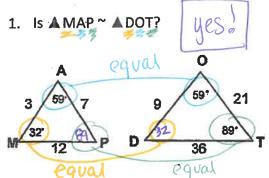
Scale factor:

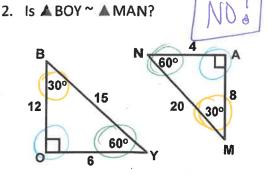
2

Missing value:

?= 4







Are all angles the same?

Are all sides proportional?

$$\frac{3}{9} = \frac{1}{3}$$
 $\frac{1}{21} = \frac{1}{3}$
 $\frac{1}{36} = \frac{1}{3}$

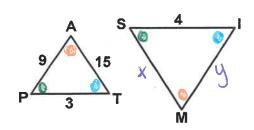
· angles equal

• Sides proportional?
$$\frac{12}{4} = \frac{3}{2}$$

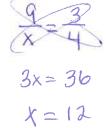
$$\frac{4}{4} = \frac{3}{2}$$

$$\frac{15}{20} = \frac{3}{4}$$

Given each pair of similar polygons, find the missing measures.



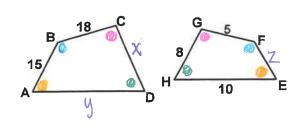
Ratios:
$$\frac{9}{x}$$
 $\frac{3}{4}$ $\frac{1}{6}$



$$3y = 15$$

 $3y = 160$
 $y = 20$

2. ABCD ~ EFGH

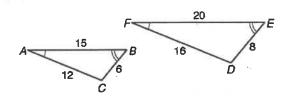


Ratios:
$$\frac{15}{Z} = \frac{18}{5}$$
 $\frac{18}{8} = \frac{x}{10}$ $\frac{18}{5} = \frac{y}{10}$ $\frac{18}{5} = \frac{y}{10}$ $\frac{18}{5} = \frac{18}{5}$ $\frac{18}{5} = \frac{y}{10}$ $\frac{18}{5} = \frac{18}{5}$ $\frac{18}{5} = \frac{18}{10}$ $\frac{18}{5} = \frac{18}{10$

To Similar Figures Practice:

Use the figure for Exercises 1 and 2. The triangles are similar.

1. Name the pairs of congruent angles.



2. Write the corresponding side lengths in the proportion below.

$$\frac{AB}{FE} = \frac{CB}{DE} = \frac{AC}{FD}$$

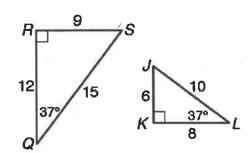
Use the figure for Exercises 3 and 4. The triangles are similar.

3. Circle the correct similarity statement.

$$\triangle QRS \sim \triangle KJL$$
 $\triangle RSQ \sim \triangle KJL$ $\triangle QSR \sim \triangle LKJ$

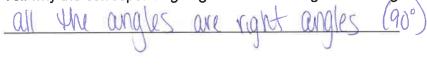
 Write a proportion with all three pairs of corresponding side lengths.

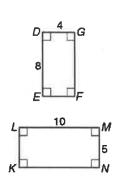
$$\frac{12}{8} = \frac{9}{6} = \frac{15}{10}$$



Use the figure for Exercises 5 and 6.

5. Tell why the corresponding angles in the rectangles are congruent.





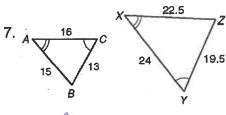
6. Substitute numbers for the side lengths and reduce each ratio to simplest form.

$$\frac{DG}{MN} = \frac{4}{5}$$

$$\frac{DE}{LM} = \frac{8}{10} = \frac{4}{5}$$

Identify the pairs of congruent corresponding angles and the Corresponding sides.



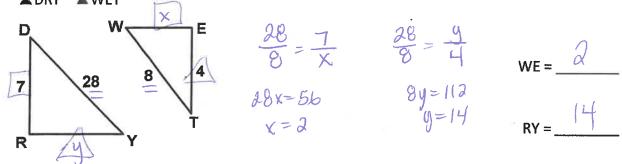






$\frac{Ab}{AB} = \frac{BC}{XZ} - \frac{AC}{XY}$ Given each pair of similar polygons, find the missing measures.

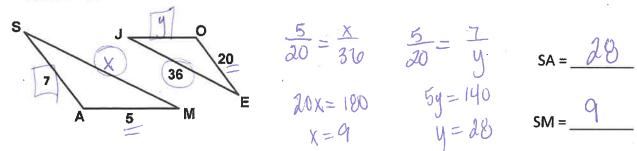




$$\frac{28}{8} = \frac{7}{x}$$

$$\begin{array}{l}
48x = 56 \\
x = 2
\end{array}$$

10. ▲ SAM ~ ▲ JOE



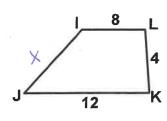
$$\frac{5}{20} = \frac{x}{310}$$

$$5y = 140$$

 $y = 28$

$$y = 28$$

11. IJKL~MNOP



$$\frac{4}{3} = \frac{x}{6}$$

$$3x = 44$$

$$4y = 24$$

$$4y = 6$$

$$\frac{4}{3} = \frac{12}{2}$$

Side-Angle-Side Similarity

SAS

Triangle Similarity

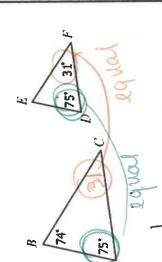
Angle-Angle Similarity AA

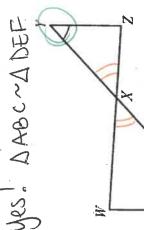
angles are congruent, then the triangles are similar P two corresponding

Determine IP the examples below are similar by ARr. P yes, write a similarity statement.



60





ត

D /x2 yes! AVXW~

SSS

Side-Side-Side Similarity

are proportional, then the IP all corresponding sides Iriangles are similar

angles are congruent, then the

triangles are similar.

if two corresponding sides are

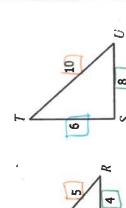
proportional and the included

are similar by SSS-. If yes, write Determine if the examples below a similarity statement.

are similar by SAS... If yes, write Determine if the examples below

a siniarity statement

ର

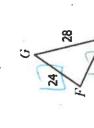


20

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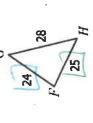
712

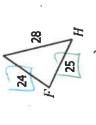
" @|9



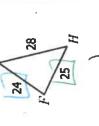
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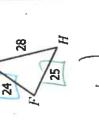
YES: AJKL ~ AMNP

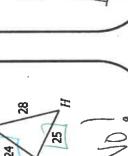




25 0







6/20

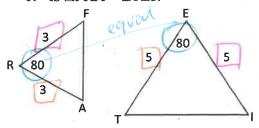


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Proving Similar Triangles Practice:

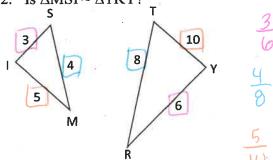
Are the following triangles similar? Why or why not?

1. Is $\triangle FRA \sim \triangle TEI$?



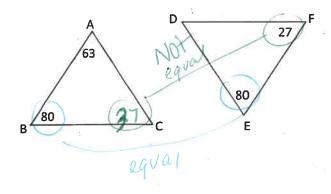
Method? SAS^
Similar? YES or NO
Scale factor (if yes) × 5/3

2. Is $\Delta MSI \sim \Delta TRY$?



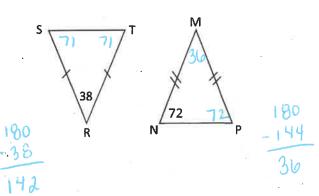
Method? Similar? YES or NO
Scale factor (if yes) × 2

3. Is $\triangle ABC \sim \triangle DEF$?



Similar? YES of NO
Scale factor (if yes)

4. Is $\triangle RST \sim \Delta MNP$

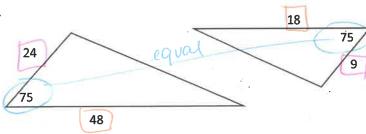


Method?

Similar? YES or NO

Scale factor (if yes)

5.

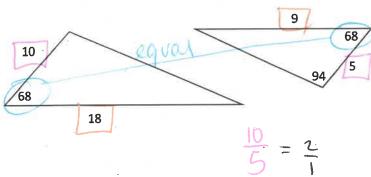


$$\frac{24}{9} = \frac{8}{3}$$

Method?

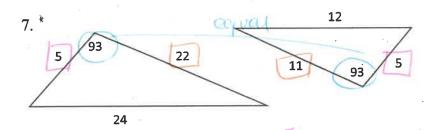
Similar? YES or NO

Scale factor (if yes) ______ x 3/9



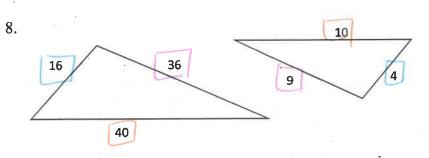
Scale factor (if yes) _____

$$\frac{10}{5} = \frac{2}{1}$$



Similar? YES or NO

 $\frac{5}{5} = \frac{1}{1}$ $\frac{22}{11} = \frac{2}{1}$ Scale factor (if yes)



Similar? YES or NO

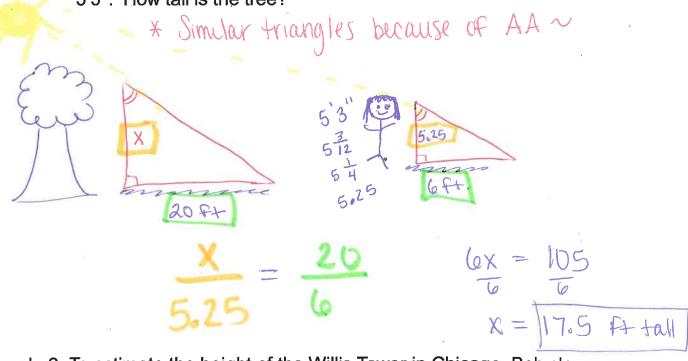
Scale factor (if yes) _____

$$\frac{36}{9} = \frac{4}{1} + \frac{40}{10} = \frac{4}{1}$$

Shadow & Mirror Application:



Example 1: While playing at the park, Katie noticed a large tree and wondered how tall that tree was. She observed that the tree had a 20 ft. shadow, while Katie's shadow was 6 ft. long. Katie knows that she is 5'3". How tall is the tree?



Example 2: To estimate the height of the Willis Tower in Chicago, Bob places a mirror 87 meters from the base of the building. Bob then steps back 0.4 meters until he sees the top of the building in the mirror. If Bob is 1.92 meters tall, how tall is the building?

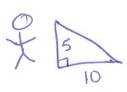
* Similar trianges because of $AA \sim$ HH
HH

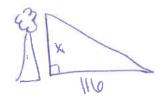
NOTICE 1.92 m $4X = \frac{107.04}{.4}$ $X = \frac{417.6}{.4}$ $X = \frac{417.6}{.4}$ $X = \frac{417.6}{.4}$

(3)

Shadow & Mirror Practice:

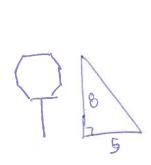
1. Jill is 5 feet tall. She casts a shadow that is 10 feet long. A nearby tree casts a shadow that is 116 feet long. How tall is the tree?

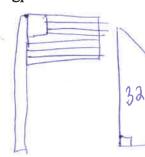




$$\frac{5}{x} = \frac{10}{116}$$

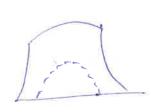
2. A sign is 8 feet high and casts a shadow that is 5 feet long. It is next to a flagpole that is 32 feet tall. How long of a shadow will the flagpole cast?

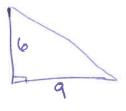




$$\frac{8}{32} = \frac{5}{X}$$

3. A 6 ft tall tent casts a 9 ft shadow. If the cardboard box next to the test casts a shadow that is 6 ft long, how tall is the box?



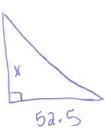


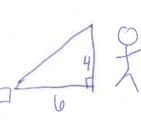


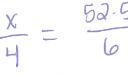


4. Tyler wants to know how tall his house is, so he puts a mirror 52.5 feet away from the house. Tyler stands back 6 feet until he can see the top of his house in the reflection of the mirror. If Tyler is 4 feet tall, then how tall is the house?



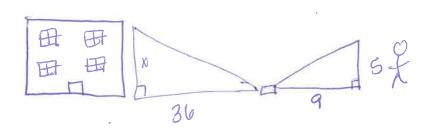






14)

5. Rianna is thinking about riding the Tower of Terror, an amusement park ride that takes its passengers straight up and then free falls. Being a bit afraid of heights, Rianna wants to know the height of the ride before she gets on. So she takes her compact mirror and places it 36 feet from the ride. She then steps back 9 feet so that she can see the top of the ride in the mirror. If Rianna is 5 feet tall, how tall is the Tower of Terror?



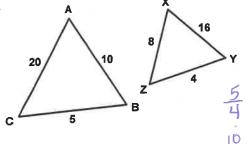
$$\frac{x}{5} = \frac{3b}{9}$$

$$x = 20 P +$$

Review:

Determine if each pair of triangles are similar. If so, state the method and write a similarity statement. Show all work!

1.

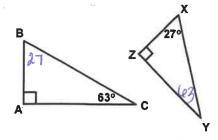


YES or NO

By which method? SSS

$$\triangle$$
 ABC \sim \triangle \times Z \checkmark

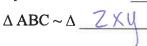
2.



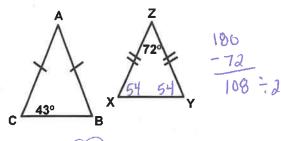
YES

YES or NO

By which method? AA



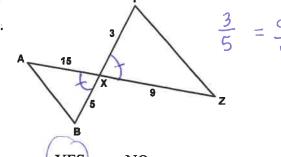
3.



YES or NO

By which method?

4.



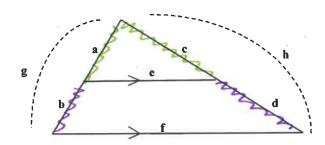
YES or NO

By which method?



F

Triangle Proportionality:



If a line is <u>Parallel</u>

_to one side of a triangle, then that line divides the two sides

proportionally

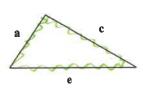
Proportion:

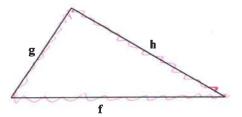
 $\frac{Q}{D} = \frac{C}{Q} \leftarrow part$

Now imagine the two triangles separated.

These are similar triangles, so their sides are also

Proportionate

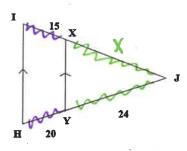




Proportions:

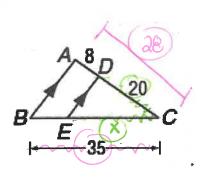
a = c = e & part f & whole

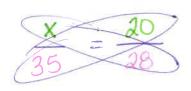




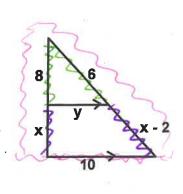
$$20x = 360$$
$$x = 18$$

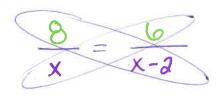
Example 2. Find CE





Example 3. Solve for x & y

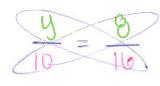




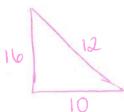
$$8(x-2) = 6x$$

 $8x - 16 = 6x$
 $-16 = -2x$
 $x = 8$





Ney=80
$$y=5$$



(1) Triangle Proportionality Practice:

Complete each proportion.

1.
$$\frac{AD}{AC} = \frac{AE}{AC}$$

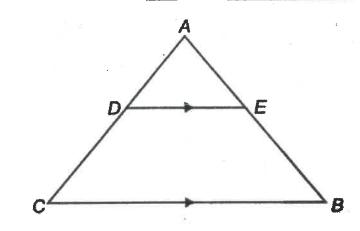
1.
$$\frac{AD}{AC} = \frac{AE}{AC}$$
 2. $\frac{AD}{DC} = \frac{AE}{EB}$

3.
$$\frac{DE}{CB} = \frac{AD}{CB}$$

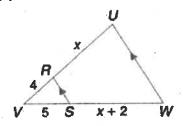
4.
$$\frac{CB}{DE} = \frac{AB}{AE}$$

5.
$$\frac{AC}{AD} = \frac{AB}{AE}$$

6.
$$\frac{DE}{CB} = \frac{AE}{AB}$$



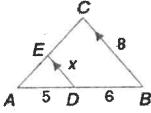
7.



Proportion:

$$\frac{4}{x} = \frac{5}{x+2} \qquad 5x = 4x+8$$

$$x = 8$$



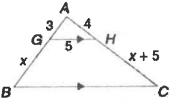
Proportion:

$$\frac{5}{11} = \frac{x}{8}$$

$$11x = 40$$

$$x = 40/11$$

8.



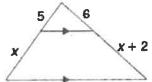
Proportion:

$$\frac{3}{x} = \frac{4}{x+5}$$
 $4x = 3x+15$

$$x = 15$$

$$HC = 20 AC = 24$$

10.

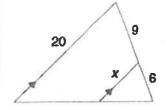


Proportion:

$$\frac{5}{X} = \frac{6}{x+2}$$

$$6x = 5x + 10$$

11.

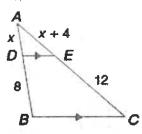


Proportion:

$$\frac{6}{15} = \frac{x}{20}$$
 $15x = 120$

$$15 \times = 120$$
$$X=8$$

12.



Proportion:

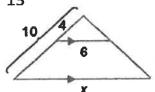
$$\frac{x}{8} = \frac{x+4}{12}$$

$$12x = 8x+32$$

$$4x = 32$$

$$AD = 10 AE = 12$$

13

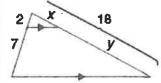


Proportion:

$$\frac{4}{10} = \frac{6}{x}$$
 $4x = 60$

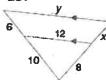
$$x = 15$$

14.



Proportion:

$$\frac{2}{9} = \frac{\times}{18}$$

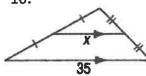


 $\frac{10}{6} = \frac{8}{x} \qquad 10x = 48$ x = 4.8

Proportion:

$$\frac{y}{12} = \frac{16}{10}$$
 $10y = 192$

16.



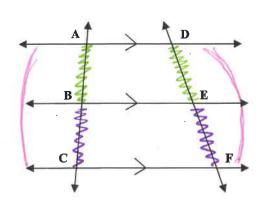
Parallel Lines and Proportions:

** When lines are ____________________, they create proportional parts.

$$\frac{\text{part}}{\text{part}} \frac{AB}{BC} = \frac{DE}{\text{EF}} \frac{\text{top}}{\text{part}} \frac{\text{part}}{\text{part}} \frac{BC}{\text{EF}} = \frac{AB}{\text{DE}} \frac{\text{part}}{\text{part}}$$

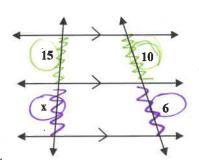
Part
$$\frac{AB}{AC} = \frac{DE}{DF}$$
 part while $\frac{DF}{AC} = \frac{EF}{BC}$ part

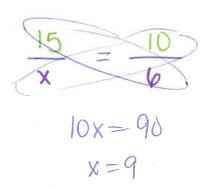
whole
$$\frac{DF}{AC} = \frac{EF}{BC}$$
 part

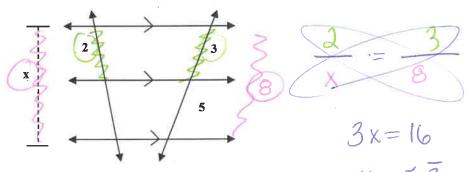


Examples

1.
$$x =$$







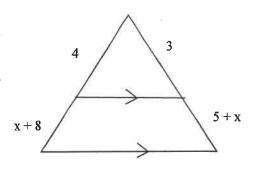
$$X = 5.\overline{3}$$
 or $5\frac{1}{3}$

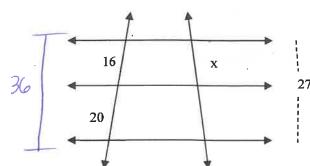
Parallel Lines and Proportions Practice:



Find each indicated measure. You must set up a proportion for each problem.

2.
$$x = \frac{12}{12}$$





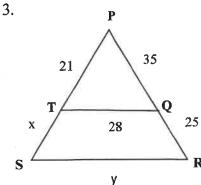
$$\frac{4}{x+0} = \frac{3}{5+x}$$

$$3x+24=20+4x$$

$$4=x$$

$$\frac{x}{27} = \frac{16}{36}$$
 $36x = 432$ $x = 12$

$$36x = 432$$



$$\frac{21}{x} = \frac{35}{25} \qquad 35x = 525$$

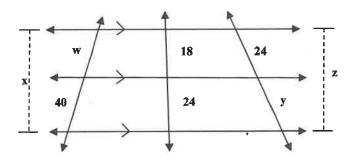
$$\frac{28}{9} = \frac{35}{60}$$

$$35k = 525$$

$$\frac{28}{9} = \frac{35}{60}$$
 $359 = 1680$ $y = 48$

$$x = \frac{15}{}$$

$$y = \frac{40}{}$$



$$\frac{18}{24} = \frac{24}{4}$$

$$18y = 576$$

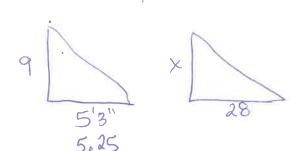
$$\mathbf{w} = 30$$

$$x = 70$$

(Y))

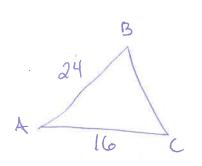
WORD PROBLEM REVIEW

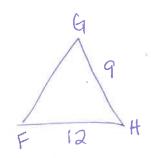
1.) When a 9-foot tall garden shed cast a 5-foot, 3-inch shadow, a house cast a 28-foot shadow. What is the height of the house? Draw a picture!



$$\frac{9}{x} = \frac{5.25}{28}$$
 $5.25 \times = 252$
 $X = 48$

- height of the house: 48 ff
- 2.) \triangle ABC \sim \triangle FGH, AB = 24 units, AC = 16 units, GH = 9 units, and FH = 12 units. Find the scale factor of \triangle ABC to \triangle FGH. Draw a picture.





scale factor: 3/4 or .75

3.) The model of a suspension bridge is 18 inches long and 2 inches tall. If the length of the actual bridge is 1650 feet, what is the height of the actual bridge?

$$model \rightarrow 18$$
 $actual > 1650 = \frac{2}{x}$

$$18 \times = 3300$$

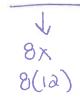
 $\chi = 183.3$

height of the actual bridge: 183.3 F+

4.) The ratio of the measures of the three angles of a triangle is 3:4:8. Find the measure of the largest angle.

$$3x + 4x + 8x = 180$$

 $15x = 180$
 $x = 180$



Largest angle: _____Q_0

Unit 6: Parallel Lines Review

Part 1: I can set up and solve proportions

1. Solve for x.
$$\frac{x}{3} = \frac{x+9}{12}$$

$$12x = 3(x+9)$$

$$12x = 3x+27$$

$$0x = 27$$

2. The scale on a map is 2 inches to 35 miles. If the distance between two cities is 152 miles, how long is the segment between the two cities on the map?

$$\frac{2 \text{ in}}{35 \text{ miles}} = \frac{x}{152 \text{ miles}}$$
 $35x = 304$
 $x = 8.7$
 $x = 8.7$
 $x = 8.7$

3. The ratio of the angles in a quadrilateral is 2:3:4:6. Find the measures of each of the angles.

add to 360
$$2(24) = 48^{\circ}$$

$$2x + 3x + 4x + 6x = 360$$

$$15x = 360$$

$$x = 24$$

$$6(24) = 144^{\circ}$$

4. The ratio of the sides of a triangle is 4:6:8. The perimeter of this triangle is 180 inches. Find the length of each side of the triangle.

6(10) = 60 in

8(10) 80 in

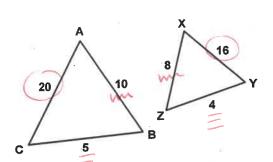
$$4x + 6x + 8x = 180$$

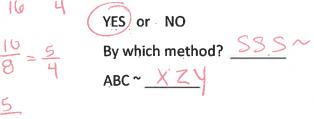
 $18x = 180$
 $x = 10$

Part 2: I understand similar figures and can prove similar triangles.

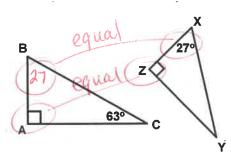
- 5. If the three sides of one triangle are <u>proport onal</u> to the three sides of another triangle, then the triangles are similar.
- 7. If two angles of one triangles are the triangles are similar. (lqual)

8. Are the following triangles similar? If so, determine by which method and write a similarity statement.

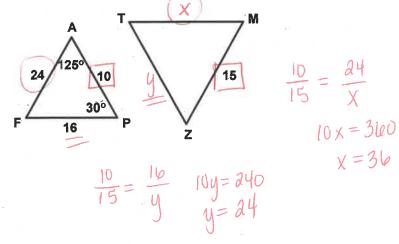




9. Are the following triangles similar? If so, determine by which method and write a similarity statement.



10. If ▲ APF ~ ▲ MZT, find each indicated measure.



$$m \angle M = 125^{\circ}$$

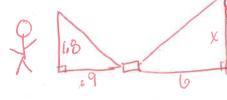
$$m \angle T = 25^{\circ}$$

$$MT = 36$$

$$TZ = 24$$

Part 3: I can use similar triangles to solve shadow and mirror problems.

11. Nick wants to measure the height of a tree. He sees the top of a tree in a mirror on the ground that is 6 meters from the tree. How tall is the tree if Nick is 0.9 meters from the mirror and his eyes are 1.8 meters from the ground?



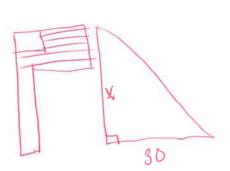
$$\frac{1.8}{X} = \frac{.9}{6}$$

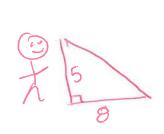
$$.9X = 10.8$$

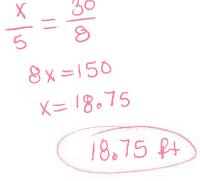
$$X = 12$$



12. Annie uses the shadow of a nearby tree to estimate the height of the flagpole. The length of the shadow of the flagpole is 30 ft. Annie's height to her eyes is 5 ft, and her shadow's length is 8 ft. What is the height of the flagpole?





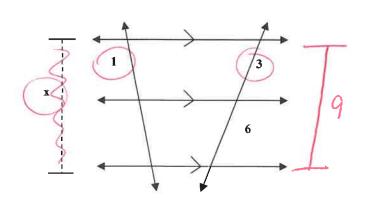


Part 4: I can set up and solve proportional parts

$$\frac{Y}{1} = \frac{9}{3}$$

$$3X = 9$$

$$X = 3$$



$$\frac{15}{10} = \frac{d}{8}$$
 $10d = 120$
 $d = 12$

$$\frac{2}{3} = \frac{x-2}{x}$$

$$2x = 3x-6$$

$$-x = -6$$

$$x = 6$$

